Core Math Tools

and Its Affordances for Mathematics Teacher Educators and for Prospective Teachers

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Session Overview

- Genesis and overview of *Core Math Tools*
- Affordances of *Core Math Tools* for Teacher Preparation
- Affordances of *Core Math Tools* for Intern Teaching
- Participant comments and questions

Common Core State Standards for Mathematics

Mathematical Practice: Use appropriate tools strategically.

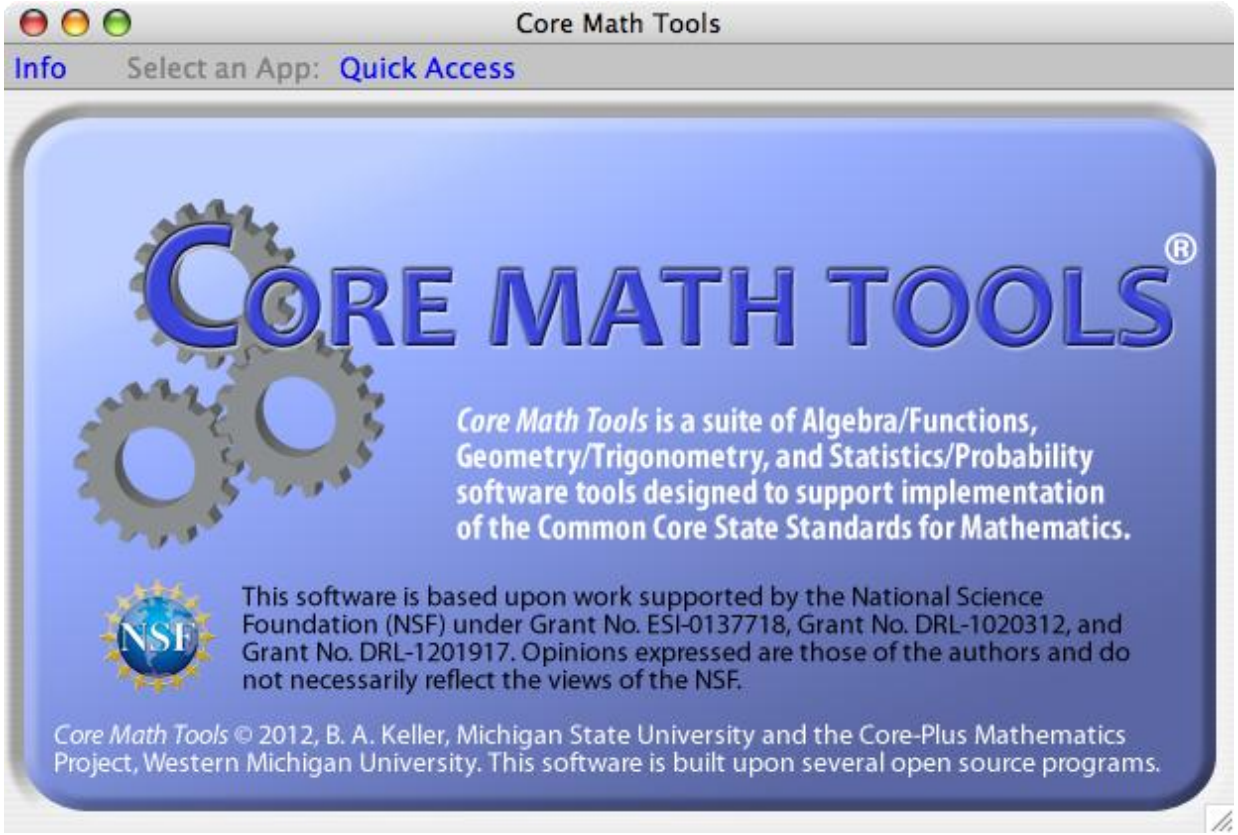
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful. . . .

Common Core State Standards for Mathematics, 2010, p. 7

Technology and Teachers' Mathematics

[Prospective] teachers should become familiar with various software programs and technology platforms, learning how to use them to analyze data, to reduce computational overhead, to build computational models of mathematical objects, and to perform mathematical experiments. The experiences should include dynamic geometry environments, computer algebra systems, and statistical software, used both to apply what students know and as tools to help them understand new mathematical ideas—in college, and in high school.

CBMS, *MET II*, 2012, p. 57


Core Math Tools
Info
Select an App:
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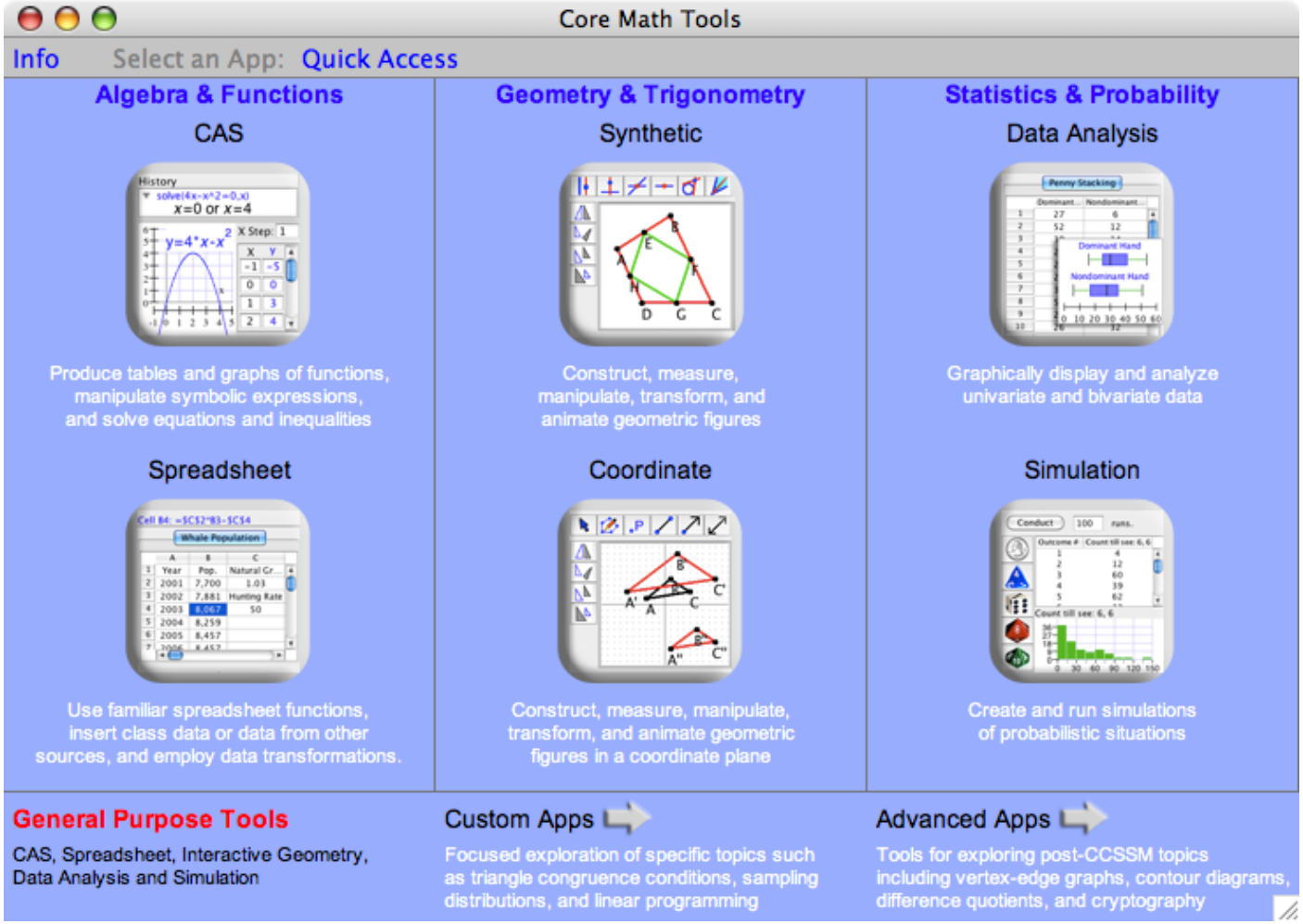
CORE MATH TOOLS®
Core Math Tools is a suite of Algebra/Functions, Geometry/Trigonometry, and Statistics/Probability software tools designed to support implementation of the Common Core State Standards for Mathematics.
NSF
This software is based upon work supported by the National Science Foundation (NSF) under Grant No. ESI-0137718, Grant No. DRL-1020312, and Grant No. DRL-1201917. Opinions expressed are those of the authors and do not necessarily reflect the views of the NSF.
Core Math Tools © 2012, B. A. Keller, Michigan State University and the Core-Plus Mathematics Project, Western Michigan University. This software is built upon several open source programs.

- *Core Math Tools* is freely available at:

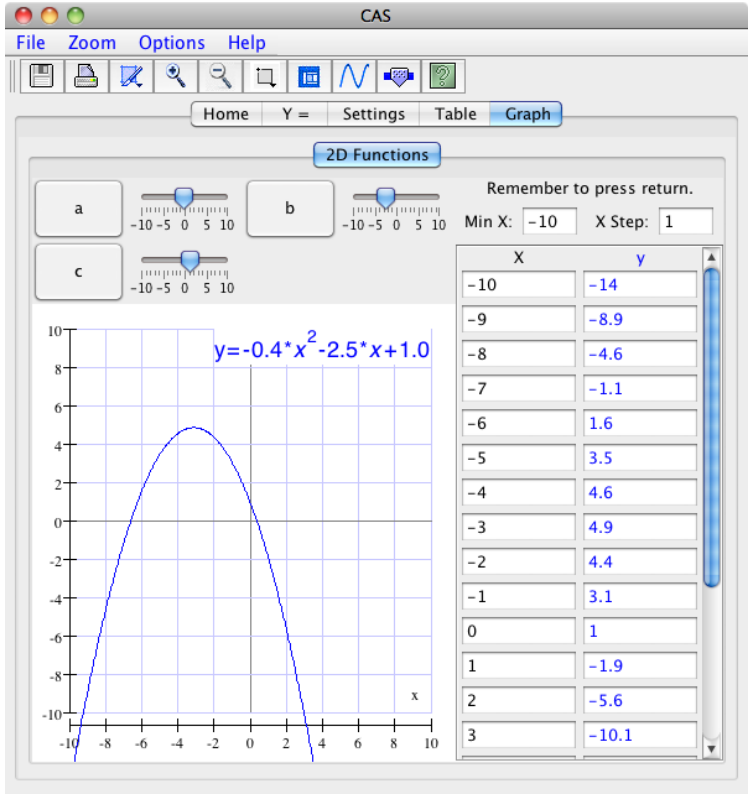
www.nctm.org/coremathtools

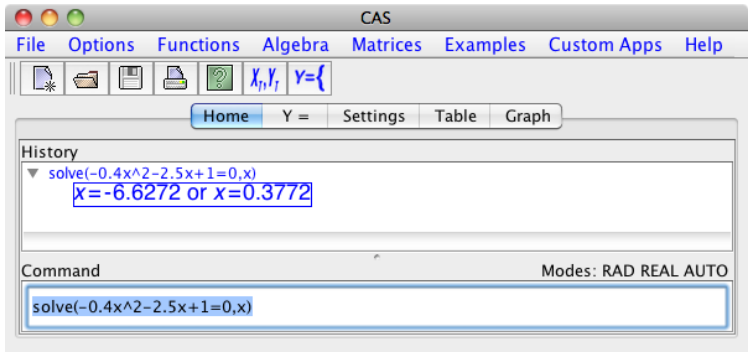
- *Core Math Tools* is accompanied by user support and resources at a CMT portal within the NCTM website.
- *Core Math Tools* is designed for use with *any* CCSSM-oriented high school textbook series.
- *Core Math Tools* is a promising resource for mathematics teacher preparation programs.



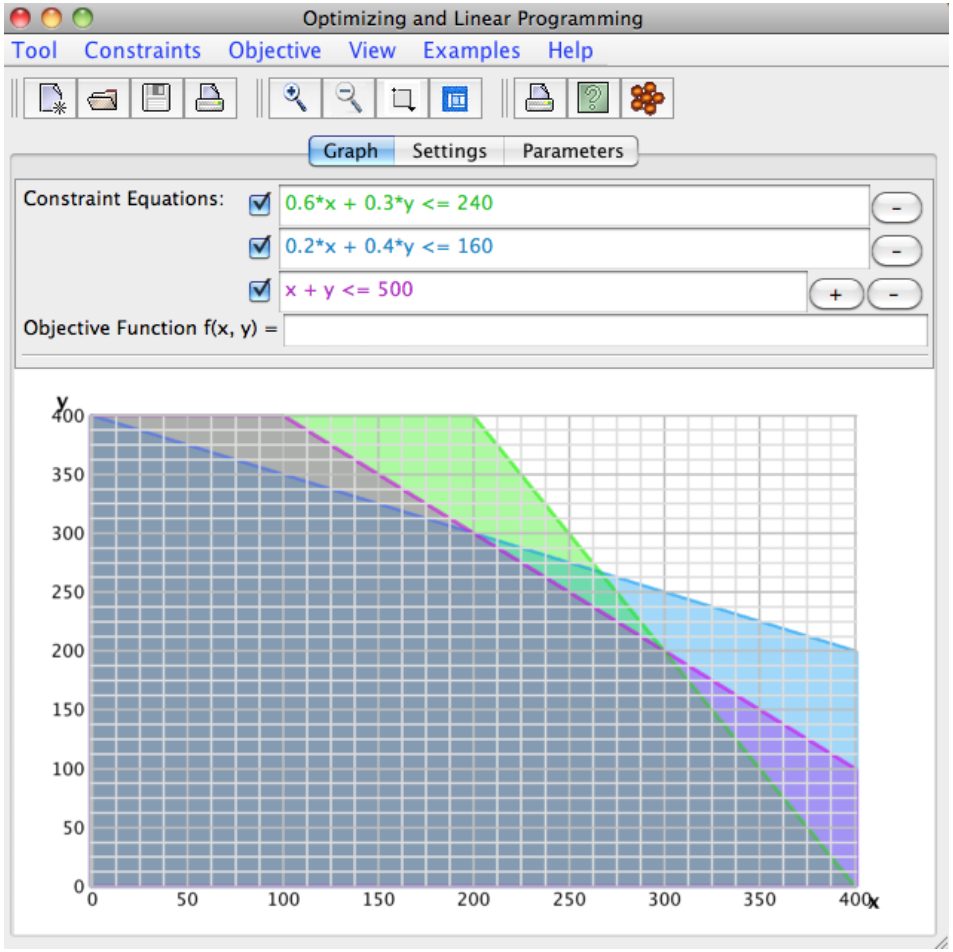
General Purpose Tools

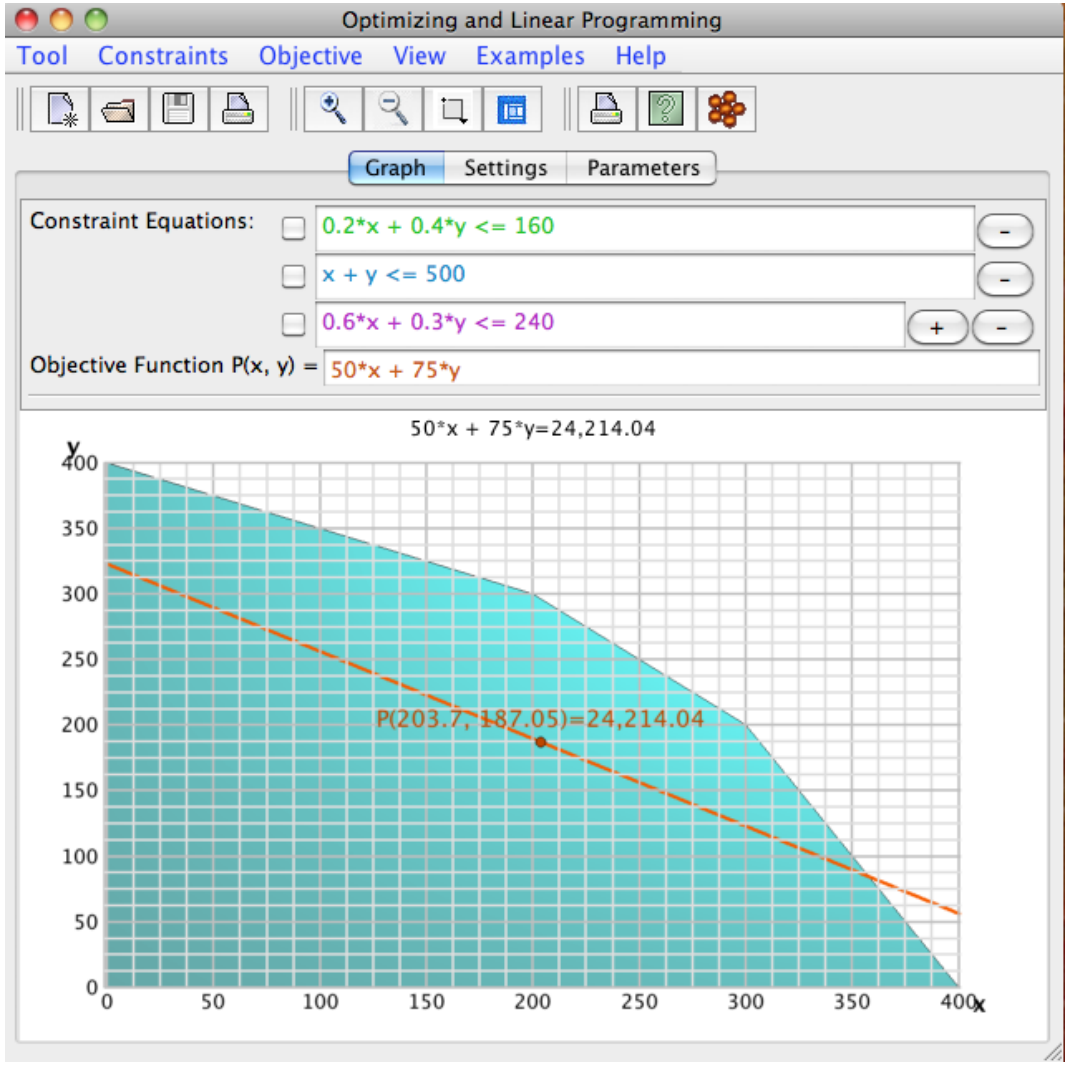
Algebra tools include an electronic spreadsheet and a computer algebra system (CAS) that produces tables and graphs of functions, manipulates algebraic expressions, and solves equations and inequalities;



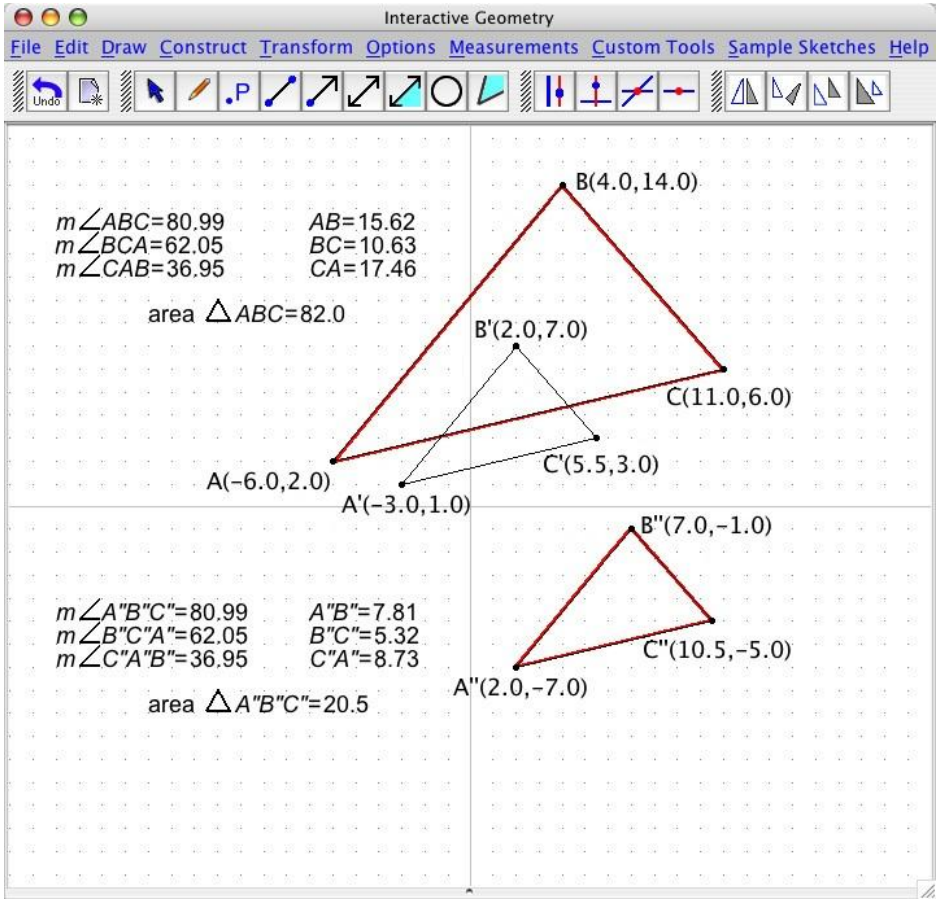


and custom apps supporting mathematical modeling.

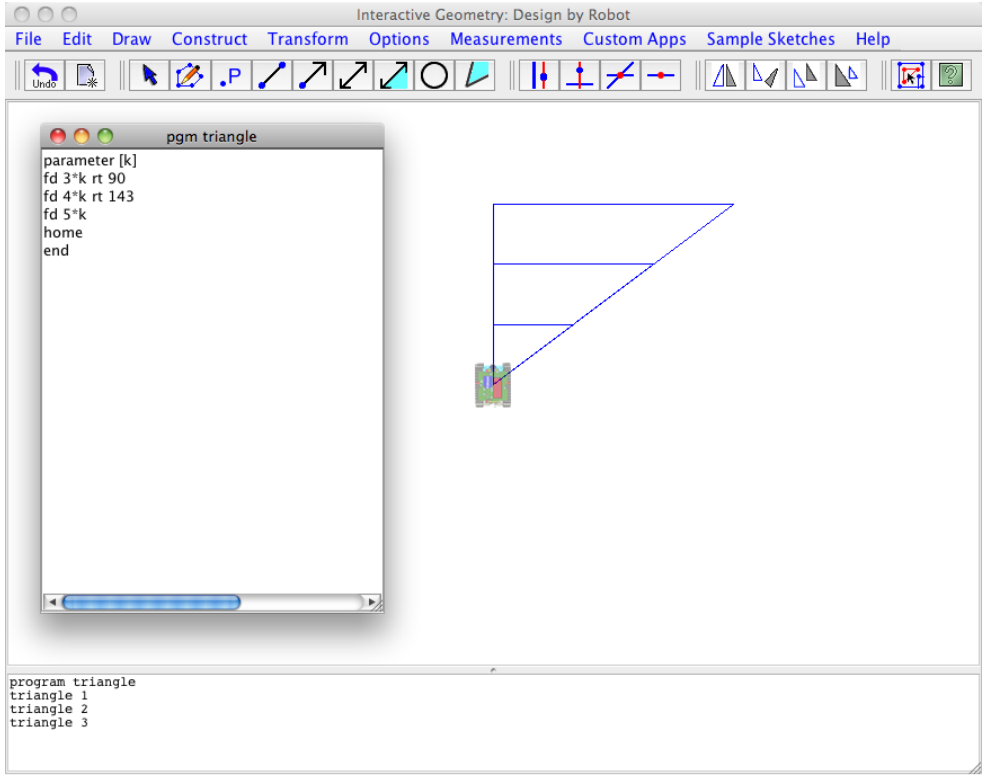



Optimizing and Linear Programming
Tool
Constraints
Objective
View
Examples
Help
Graph
Settings
Parameters
Constraint Equations:
0.2*x + 0.4*y <= 160
x + y <= 500
0.6*x + 0.3*y <= 240
Objective Function P(x, y) =
50*x + 75*y
50*x + 75*y=24,214.04
P(203.7, 187.05)=24,214.04
y
x
0
50
100
150
200
250
300
350
400

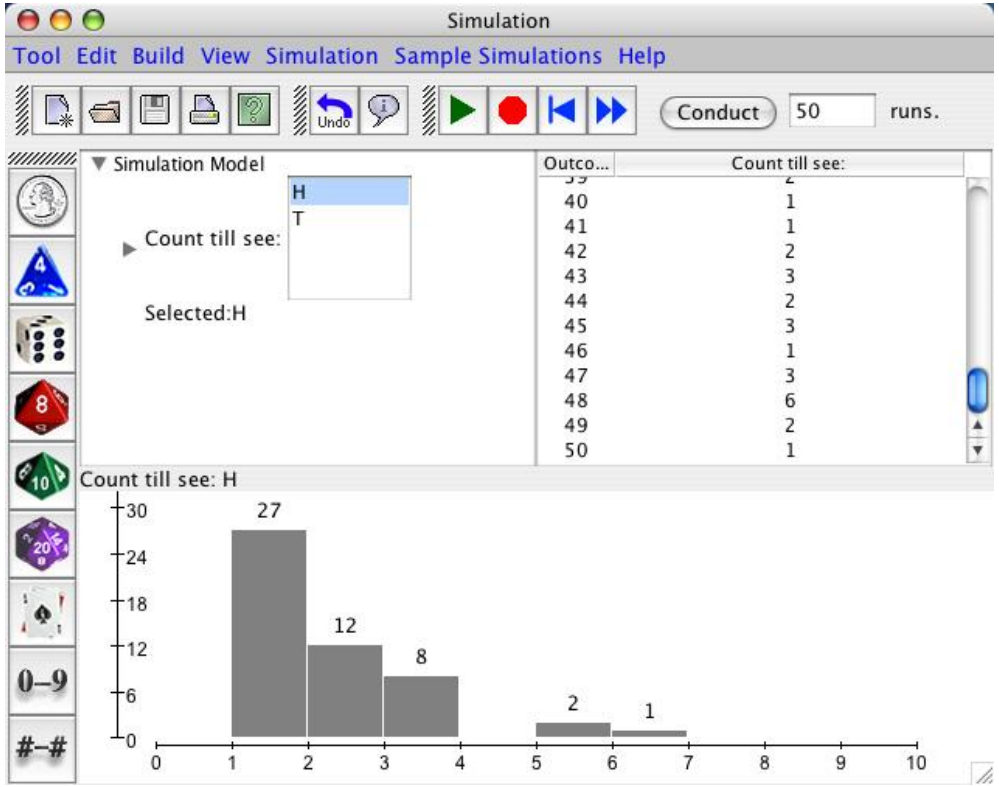
Geometry tools include an interactive drawing tool for constructing, measuring, manipulating, and transforming geometric figures in both a coordinate and coordinate-free environment, a simple object-oriented programming language for creating animations,



and custom apps for exploring a Logo-like environment, geometric models of contextual situations, design of physical mechanisms, and tilings of a plane.



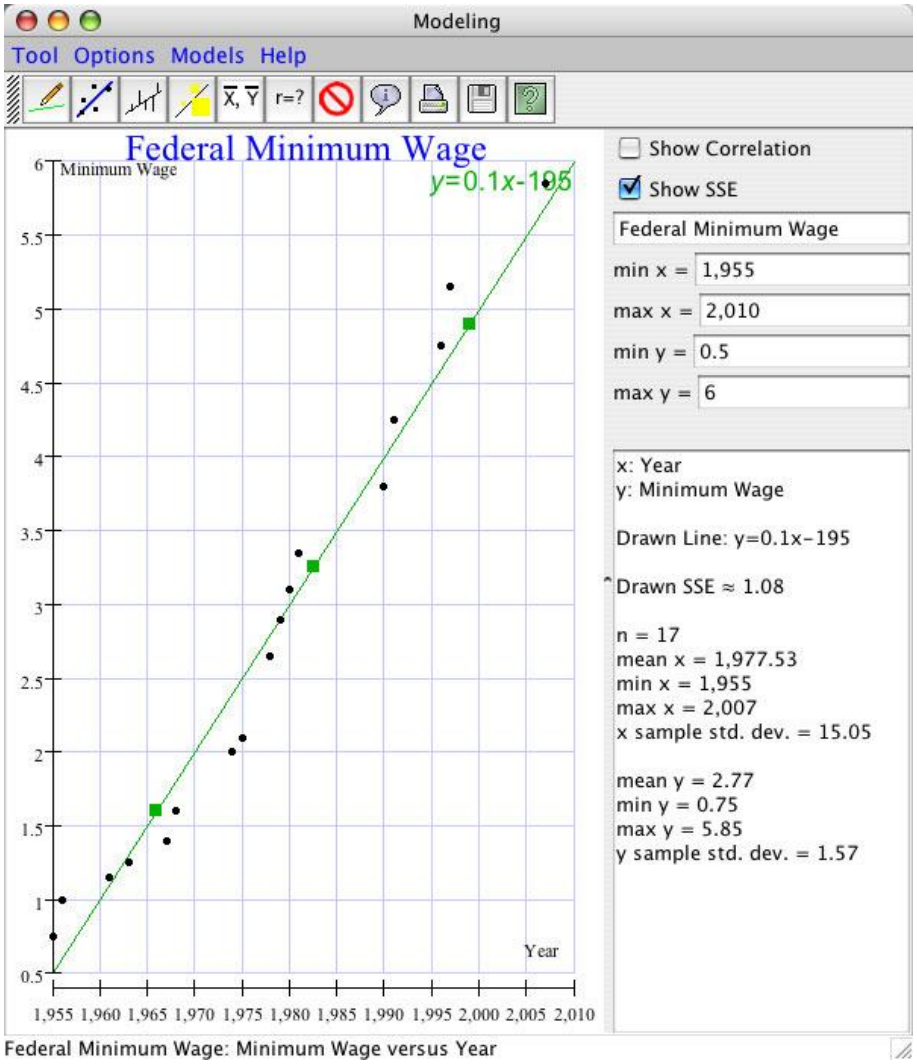
Statistics tools include tools for graphic display and analysis of univariate and bivariate data, simulation of probabilistic situations

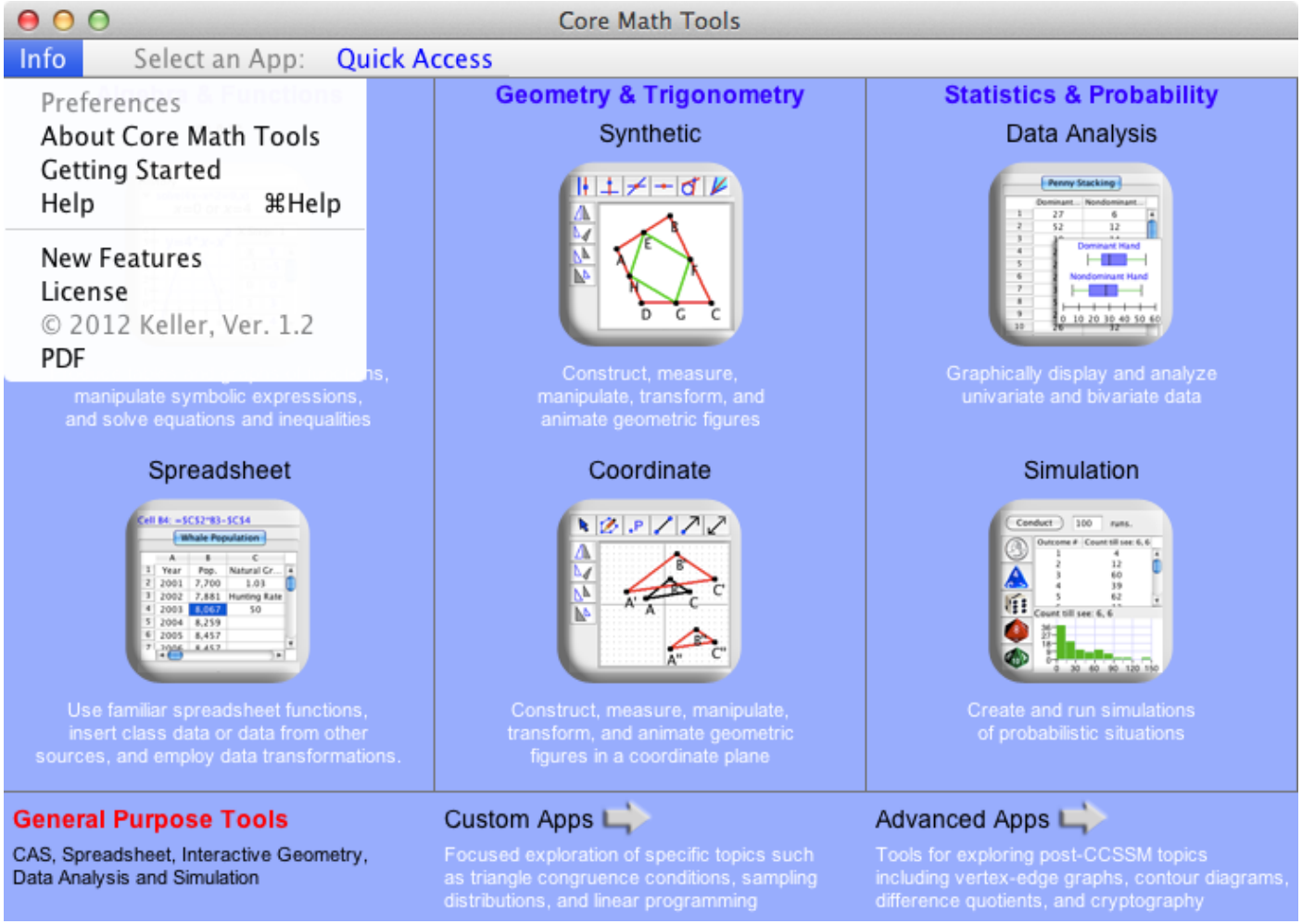


and mathematical modeling of quantitative relationships.

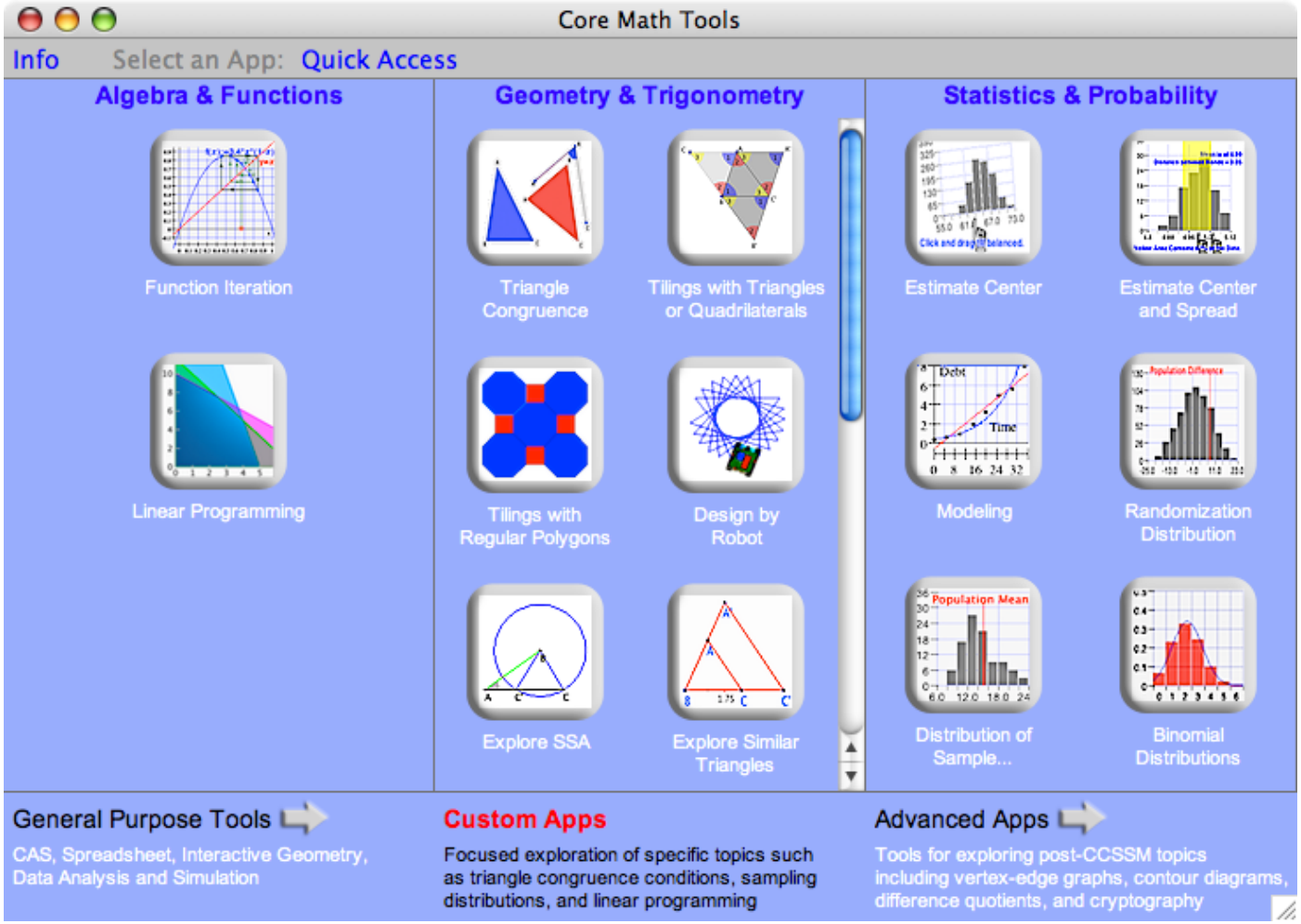
Spreadsheets allow easy insert of class data or data available from other sources.

CMT includes pre-loaded data sets for developing key statistical ideas.





CMT Resources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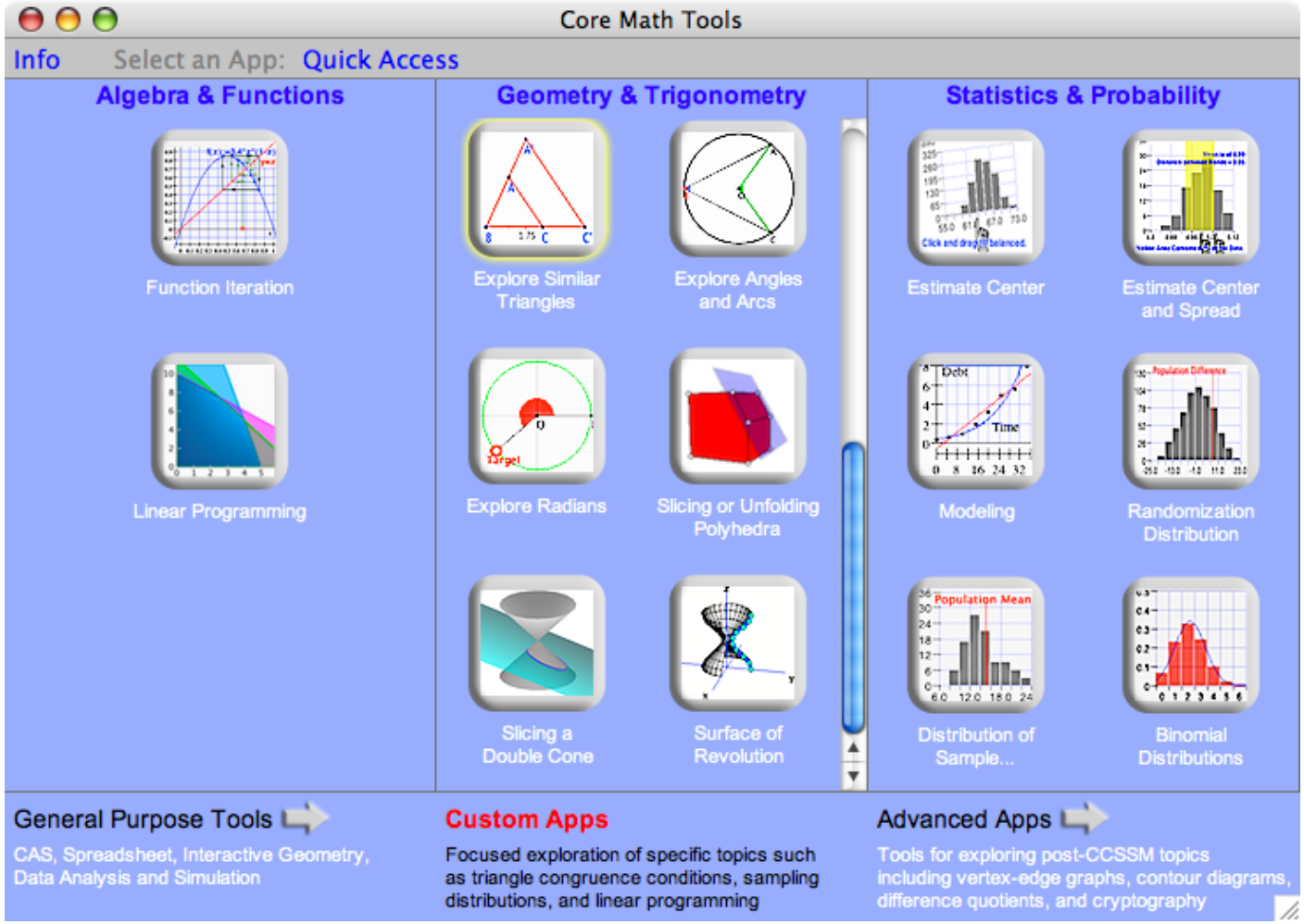
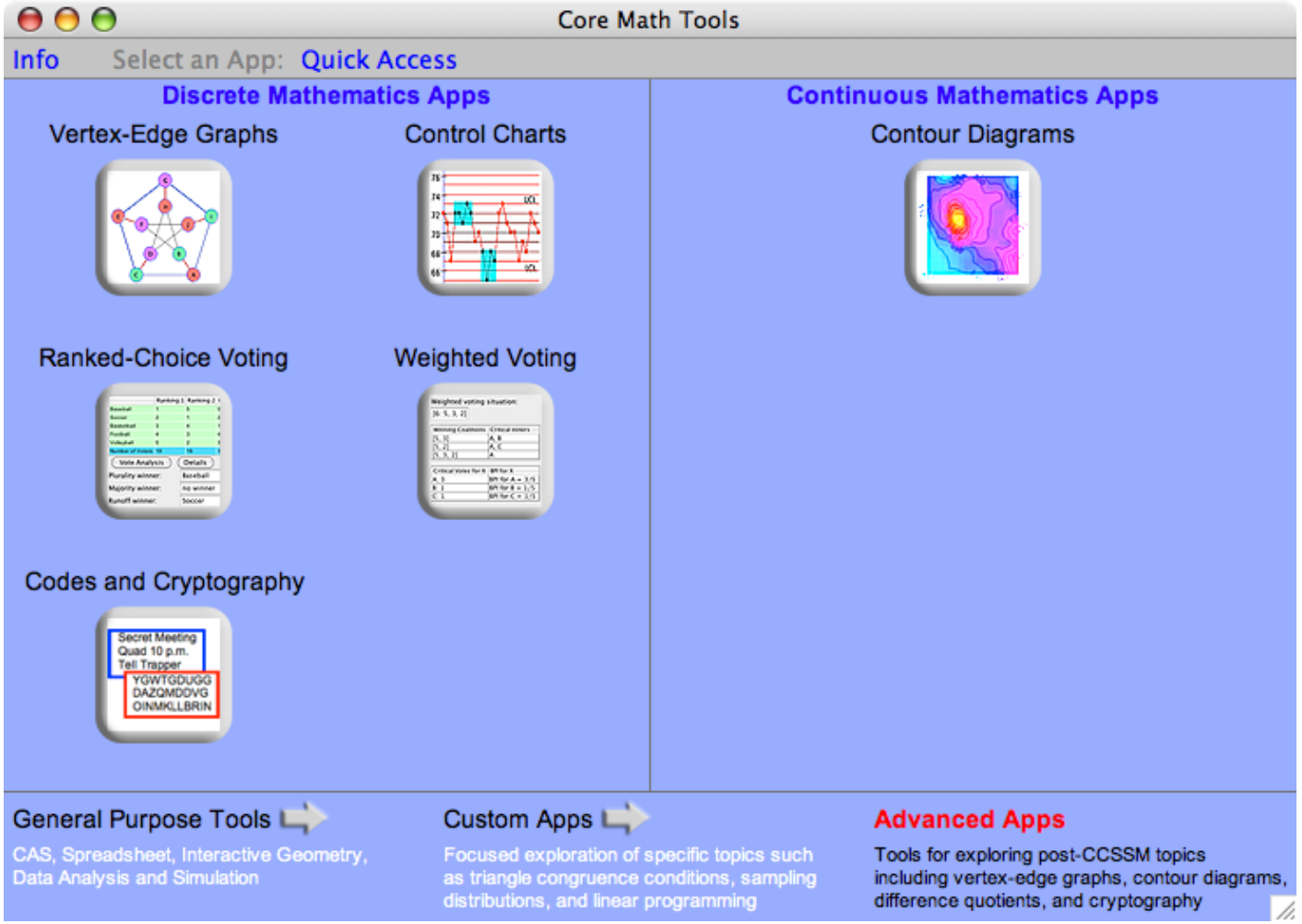
Linear Programming
Geometry & Trigonometry
Triangle Congruence
Tilings with Triangles or Quadrilaterals
Tilings with Regular Polygons
Design by Robot
Explore SSA
Explore Similar Triangles
Statistics & Probability
Estimate Center
Estimate Center and Spread
Modeling
Randomization Distribution
Distribution of Sample...
Binomial Distributions
General Purpose Tools
CAS, Spreadsheet, Interactive Geometry, Data Analysis and Simulation
Custom Apps
Focused exploration of specific topics such as triangle congruence conditions, sampling distributions, and linear programming
Advanced Apps
Tools for exploring post-CCSSM topics including vertex-edge graphs, contour diagrams, difference quotients, and cryptography

Custom Apps



More Custom Apps



Advanced Apps

Three Case Descriptions of CMT Use in Mathematics Teacher Education Programs . . .

Building Preservice Candidates' TPCK Using Core Math Tools

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Auburn University Secondary Mathematics Program

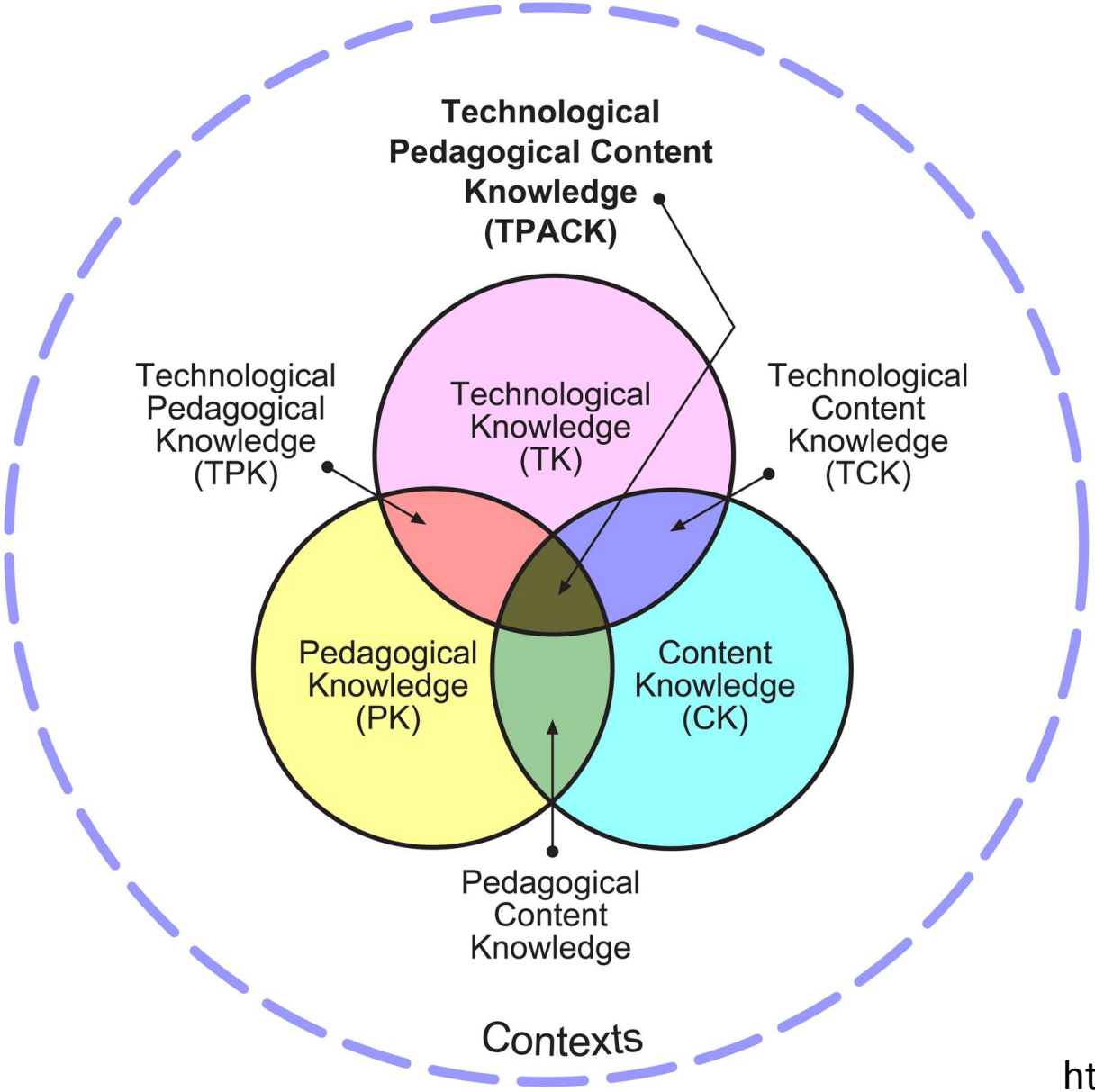
- Core Curriculum (38 hours)
- Mathematics (42 hours)
- Educational Foundations (12-14 hours)
- Mathematics Education (12-14 hours)
  - Technology and Applications in Secondary Mathematics
  - Middle School Mathematics Methods
  - High School Mathematics Methods
  - Classroom Management (Math-specific or General)
- Internship (12 hours)

CTSE 5040 -- Technology and Applications in Secondary Mathematics Education

- Course Description: “Use of technological tools to enhance mathematics teaching and learning”
- 4 credit hours, including 15 hours of field experiences
- Class meets in computer lab – approximately 32 periods
- Students sign out an iPad 2 for use throughout the semester

Course Objectives

- To build students' understanding of:
  - Use of technology to solve mathematical problems
    - Mathematics as a meaningful activity (processes/practices)
  - Use of technology to build student mathematical understanding
    - "Mathematical Action Technologies" (Dick & Hollebrands, 2011, p. xi)
  - General use of technology to support instruction



http://tpack.org

Course Outline

- Algebra
  - WolframAlpha, Graphing Calculators, Excel
- Geometry
  - PTMT Geometry (Hollebrands & Lee, 2012) -- GSP
- Statistics
  - PTMT Statistics (Lee et al., 2010) – TinkerPlots, Fathom, Excel
- Putting the pieces together
  - Core Math Tools

CMT Outline

- Brief introduction to the suite using lesson plans from www.nctm.org/CoreMathTools/
  - Synthetic Geometry (transformations lesson; repeat constructions done earlier)
  - Data Analysis (repeat analysis of a data set from earlier in the semester)
  - Simulation (blood donor lessons)
  - Spreadsheet; CAS (flag pole lesson using each)
  - Free play with other apps

Data Analysis

- Pre-service teachers collected data on their resting heart rates.

  They then jumped in place for 1 minute, and again found their heart rates.

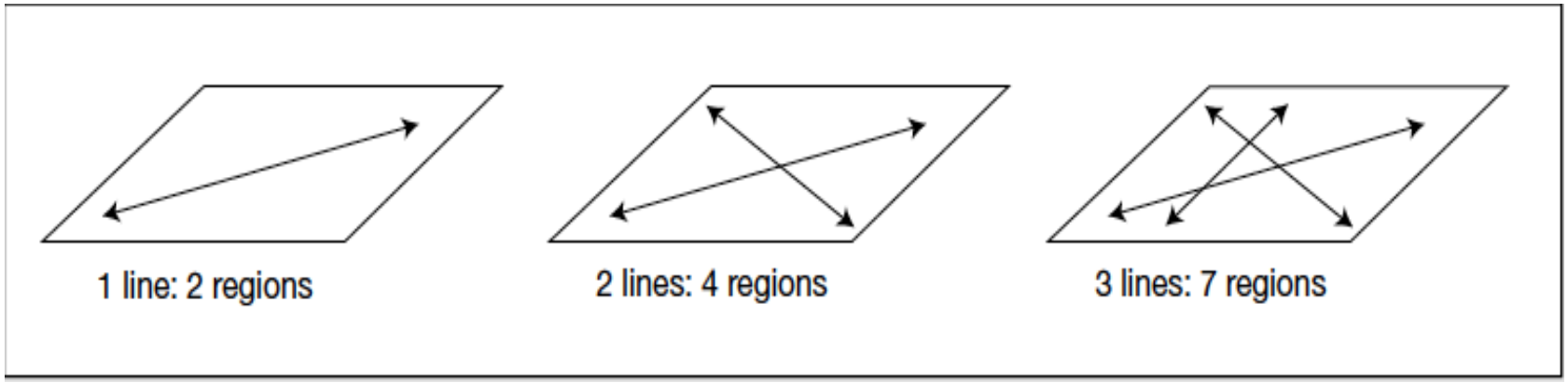
- Exploration with CMT:
  - Graphs
  - Statistics
  - Randomization distribution
  - Scatterplot – linear regression

Final Assignment

- Problems in which:
  - A variety of technological tools might be useful
  - In fact, successful solutions might use a combination of tools
  - Students were encouraged to use any tool they wished, although they were encouraged to incorporate the use of CMT
  - Students worked in groups over 4-5 class periods and presented their solutions as part of the final exam.

Patterns, Plane and Symbol

- Develop a symbolic representation for a function that produces the number of regions in a plane formed by intersecting lines such that no two lines are parallel and no more than two lines intersect in the same point, as shown in the figure.



(NCTM, 2009)

Student Solutions

Student Reactions

- Overwhelmingly positive.
  - They liked the range of tools available all in one place.
  - They felt they were useful and usable.
  - They were impressed that this was being offered by NCTM at no cost.
- There were some concerns about some apps not being user-friendly.
  - As one student noted, some of this might reflect their previous experiences with GSP and Excel.
- They particularly enjoyed the Simulation, CAS, and Data Analysis apps.
- "I also like that in most all of these tools there is one tab up in the menu that has examples, such as the blood type which we were looking at."
- Overall, they felt it was a tool that would be useful to teachers, and that they will use it.
- Several mentioned that they planned to use it in upcoming lessons for their field experiences.

Final Reflections

- CMT was a very useful tool for use in a methods class
  - Relatively low overhead to engage students
  - Accessibility means they could incorporate it in their field placements
- Using CMT as part of a summative experience vs. using it throughout the semester

Core Math Tools at Penn State
Fall 2012

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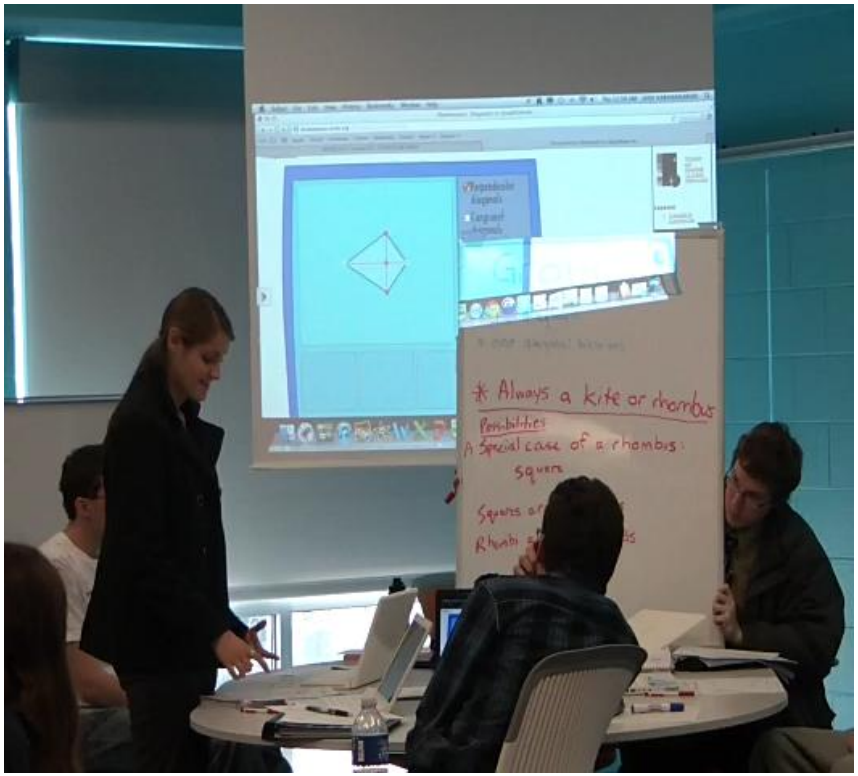
MTHED 427 Course Context

- MTHED 427 Teaching Mathematics in Technology-Intensive Environments
  - 1 of 2 mathematics education courses in the 1st of 3 semesters
  - Developed from programming focus in 1980s through tools to teaching
- Technology?
  - Broad definition of technology (e.g., laptops, manipulatives, applets, pencil)
  - Mathematics technology and communication/collaboration technology
- Focus?
  - Not prioritize developing fluency with today's products
  - Be open to using today's tools, be ready to learn to use unfamiliar and new tools, and expect to teach with a variety of technology genres
  - Approach teaching with today's technology given a sense of where things have been and a wild anticipation for the future

Teacher Learning & Collaboration

- Some atypical elements
  - Learn Lab setting (collaboration focus)
  - Collaboration with local middle school (GeoGebra)
- Focus is on learning to teach with technology and to teach each other and learn together.
- Without replicating some favorites…



1. CMT and Asymptotes

- Sheri, the lone master's student in the group, wanted others to encounter asymptotes as much more than "lines that the graph approaches and doesn't cross."

  Notes:

  The ideas that follow came from her tutoring experience, study of textbooks, and her curiosity.

  It requires more than "solve" in CAS.

Sheri's Question to the Group

Can a curve cross an asymptote?

Sheri's Question to the Group

Can asymptotes be curves?

Sample from Sheri's Chart

	$y = \dfrac{n(x)}{d(x)}$	$y = q(x) + \dfrac{r(x)}{d(x)}$	Asymptotes	x-value of asymptote intersection(s)
1	$y = \dfrac{1}{x+2}$			
4	$y = \dfrac{(x+1)^2}{x+2}$			
5	$y = \dfrac{2x^2 + x + 1}{x^2}$			
8	$y = \dfrac{2x^5 - x^2 + x - 1}{x^2}$			

Remainder

Quotient

Divisor

13/4 interpreted as
13 ÷ 4
or 3 R1
treated as
3 + ¼

$(2x^2 + x + 1)/x^2$ interpreted as
$(2x^2 + x + 1) \div x^2$
or 2 R $(x + 1)$
treated as
$2 + (x + 1)/x^2$

$$\begin{array}{r} 2 \\ x^2 \overline{)\,2x^2 + x + 1} \\ x + 1 \end{array}$$

expand((2x^2 + x + 1)/(x^2))

CMT Examples of CAS

```
expand(1/(x+2))
```

$$\frac{1}{x+2}$$

```
expand((x+1)^2/(x+2))
```

$$x+\frac{1}{x+2}$$

```
expand((2x^2+x+1)/x^2)
```

$$2+\frac{x+1}{x^2}$$

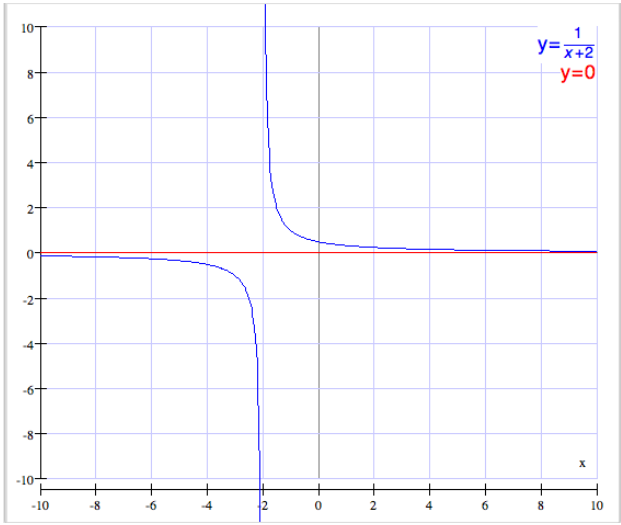
```
expand((2x^5-x^2+x-1)/x^2)
```

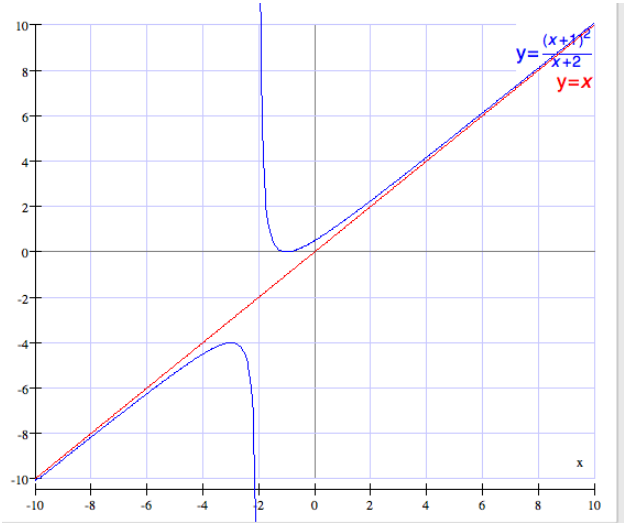
$$2*x^3-1+\frac{x-1}{x^2}$$

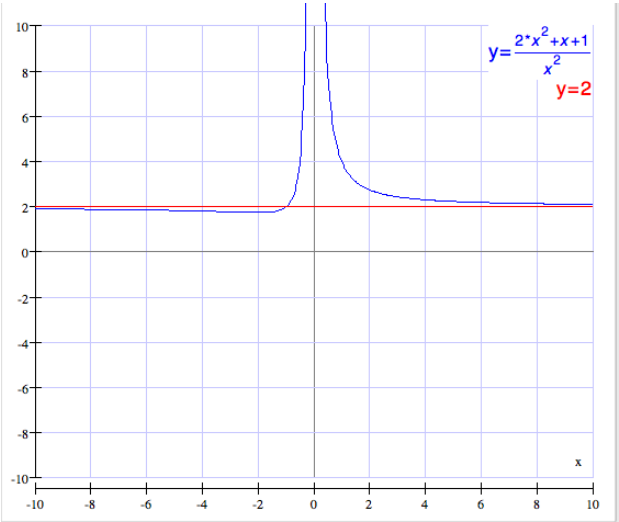
```
Done
```

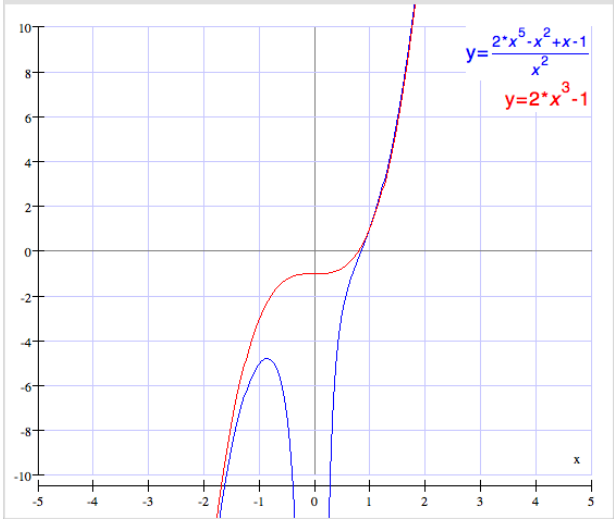
Done

CMT Examples of Graphs





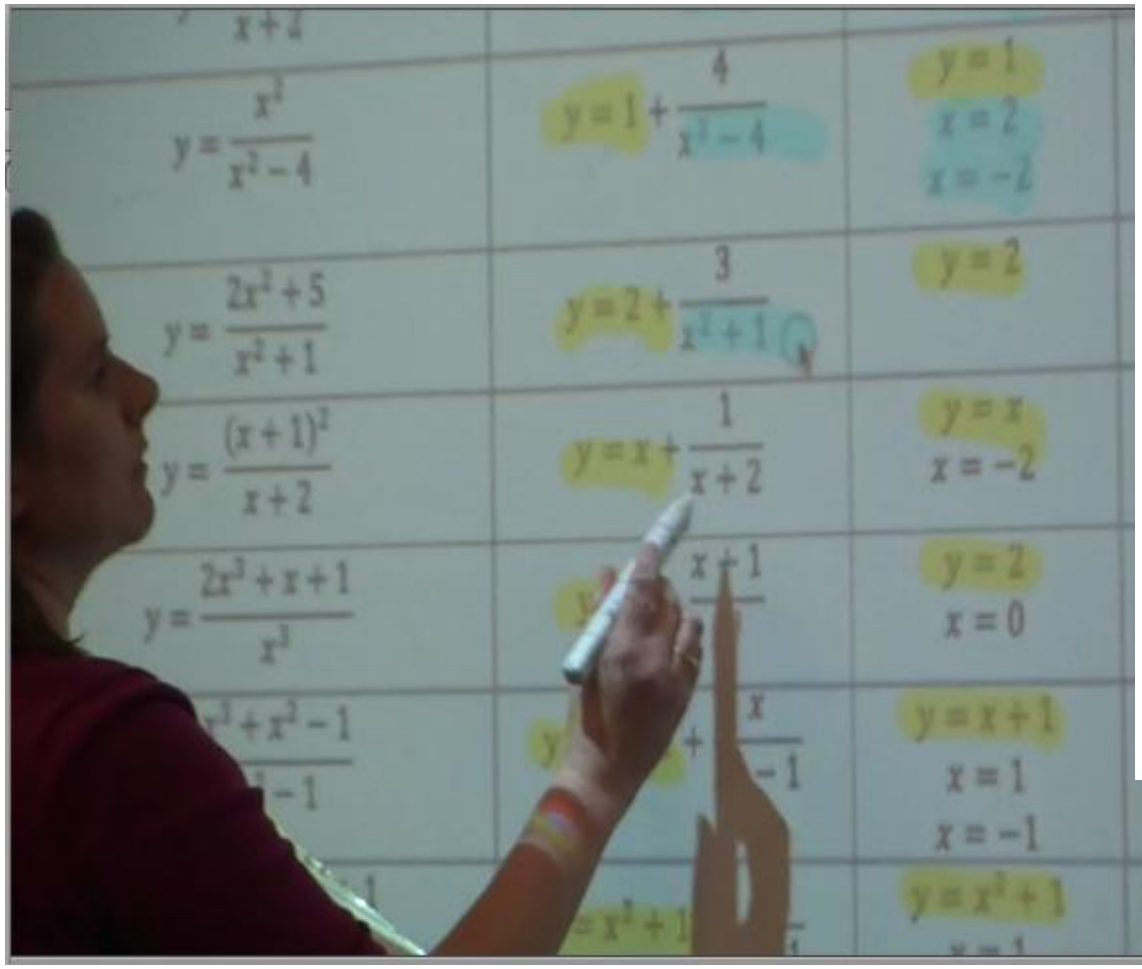




Sample from Sheri's Chart

	$y = \frac{n(x)}{d(x)}$	$y = q(x) + \frac{r(x)}{d(x)}$	Asymptotes	x-value of asymptote intersection(s)
1	$y = \frac{1}{x+2}$	$y = 0 + \frac{1}{x+2}$	$y = 0$ $x = -2$	None
4	$y = \frac{(x+1)^2}{x+2}$	$y = x + \frac{1}{x+2}$	$y = x$ $x = -2$	None
5	$y = \frac{2x^2 + x + 1}{x^2}$	$y = 2 + \frac{x+1}{x^2}$	$y = 2$ $x = 0$	-1
8	$y = \frac{2x^5 - x^2 + x - 1}{x^2}$	$y = 2x^3 - 1 + \frac{x-1}{x^2}$	$y = 2x^3 - 1$ $x = 0$	1

Patterns in Chart



Justification of these patterns was part of the post-exploration discussion!

(Students do more than report what they see on a screen.)

CMT Problem and Use

- What happened when Sheri tried #9?

$$y = \frac{-x^4 - 6x^3 - 11x^2 - 11x - 4}{(x+2)^3}$$

```
expand((-x^4-6x^3-11x^2-11x-4)/((x+2)*(x+2)*(x+2)))
```

$$\frac{-x^4}{x^3+6*x^2+12*x+8} - \left(6 + \frac{-36*x^2-72*x-48}{x^3+6*x^2+12*x+8}\right) - \frac{11*x^2}{x^3+6*x^2+12*x+8} - \frac{11*x}{x^3+6*x^2+12*x+8} - \frac{4}{x^3+6*x^2+12*x+8}$$

Points About Asymptotes Activity

Learning

- A new view of asymptotes matters.
- Seeing these relationships symbolically is a new experience. Justifying them is important.

Technology

- CMT helped with symbolic manipulation and graph production as learners generalize and justify.

2. CMT for Equation Solving

- Erica's lesson inspired by her questions:
  - What does it mean to *solve an equation*, such as $3x + 7 = 3 - x$?
  - Why do we need to pay attention to "do the same thing to both sides?"
  - What are *equivalent equations*?

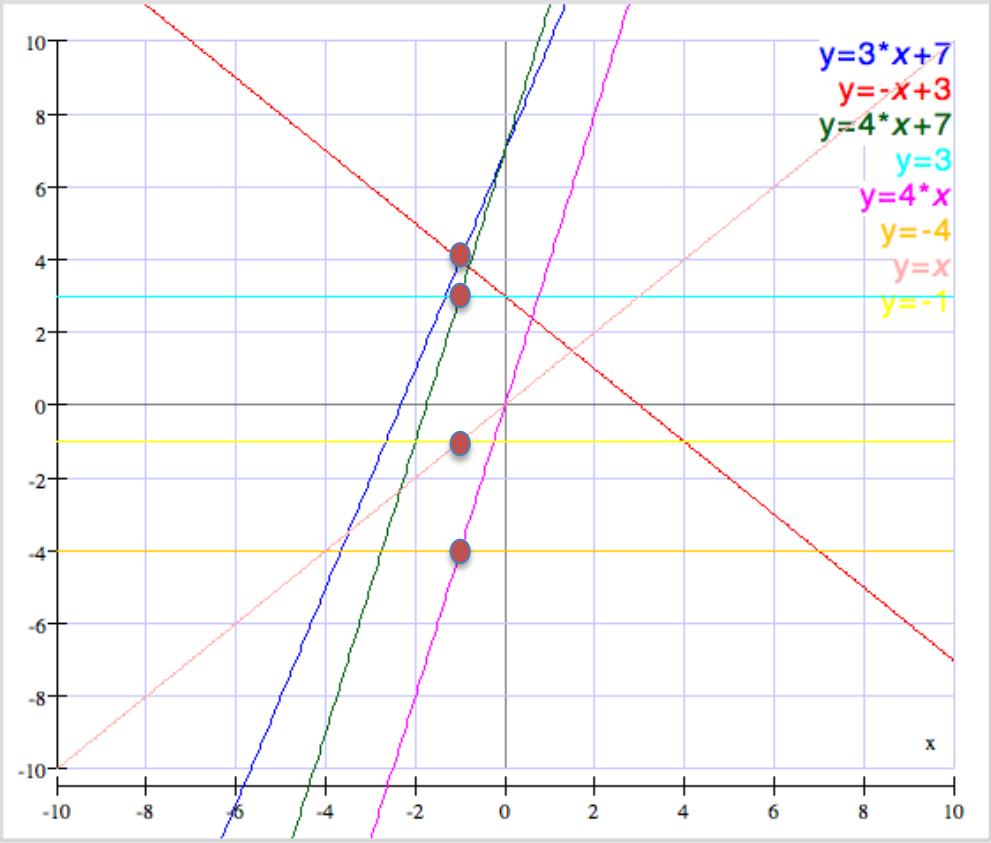
Source: Zbiek & Heid, 2011

CMT Work Sample for Erica's Task

Symbolic Work

- 3x+7=-x + 3
  $3*x+7=3-x$
- (3*x+7=3-x)+x
  $4*x+7=3$
- (4*x+7=3)-7
  $4*x=-4$
- (4*x=-4)/4
  $x=-1$

Graphs



Points About Equation Solving

Learning

- Procedural ease can be complemented by conceptual challenge.
- Structure matters—in this case, we have properties of equality and field properties.

Technology

- Use of CMT to learn a procedure by delegating the work to CMT and using multiple representations to come back to the procedure.

3. CMT for CCSSM and Parameters

- What are the effects of changing the values of *a*, *b*, *c*, and *d* on graphs of the form

$$f(x) = a \bullet (p(b \bullet (x - c))) + d \text{ ?}$$

Note:

p *represents a primitive function and not a numeric parameter;*

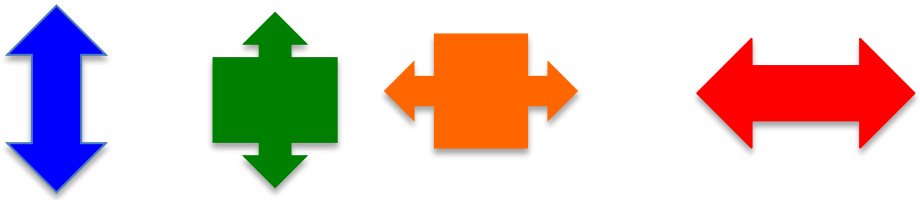
a, b, c, *and* d *are parameters.*

Source: Zbiek & Heid, 2011

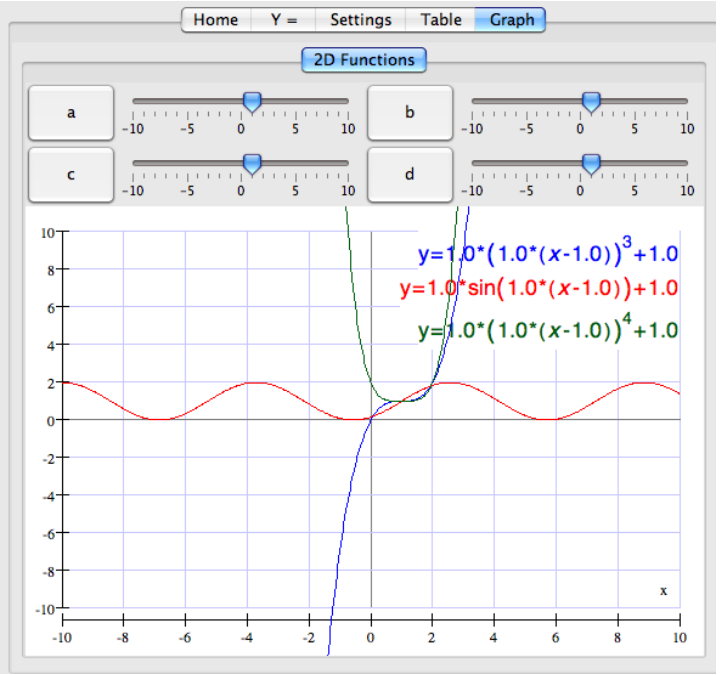
Parameters and CCSSM Content

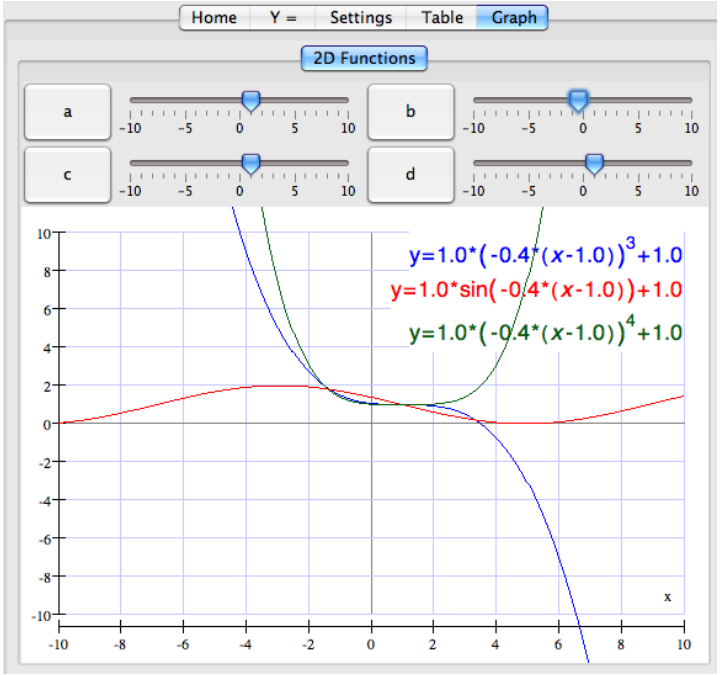
F-BF.3.

- Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. …



Different Primitive Functions





Parameters
and
Mathematical Practices

MP1.

- Make sense of problems and persevere in solving them.

MP7.

- Look for and make use of structure.

Points About Parameters

Learning

- The power of a generalized means to adjust “any” graph is useful (to teachers!).
- We understand a standard and its fit with a bigger idea.

Technology

- Sliders are easily done in CMT.
- The capability to copy and paste CMT images into other documents is valuable.

Other CMT Examples (PAMTE)

Task ideas connected to CCSSM with technology points:

- Exponents
  - Helping struggling students to learn properties

    (Based on Swierczek & Indeck, 2012)
- Arcavi's problem
  - Developing a function with reasoning beyond curve fitting

    (Based on Arcavi & Hadas, 2000; Revised CMT online example)
- Reflect a graph across a line
  - Generalizing results of reflecting a graph

    (Based on Thomas idea in Heid, Thomas, & Zbiek [in press])

Core Math Tools at University of Wisconsin - Milwaukee

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- Use of CMT in class
- CMT lesson assignment
- Video of lesson presentation
- Video of intern teacher using CMT with 8th graders

Program at UWM

Two methods courses:

Teaching Mathematics: Middle School (Summer)

Teaching Mathematics: High School (Fall)

Fall semester: ½ day Middle School student teaching

Spring semester: full day High School placement

Introduction of CMT

Core Math Tools introduced during middle school methods course

- Coordinate Geometry – Transformations
  - Properties of Reflection and Rotations
- Data Analysis – Single Variable Quantitative data
  - Histograms and Box plots
  - One variable statistics
    - Mean, Median, MAD (Mean absolute deviation – using spreadsheet)

Use of CMT
High School Methods Course

Continue “how-to”

- CAS
  - Algebra: Simplify, expand, factor, solve
- Geometry
  - Synthetic: Draw, Construct, Measure
- Data Analysis
  - Two Quantitative variables
    - Least Squares
    - Correlation
- Simulation

Problem-Solving Using CMT

Use of CAS

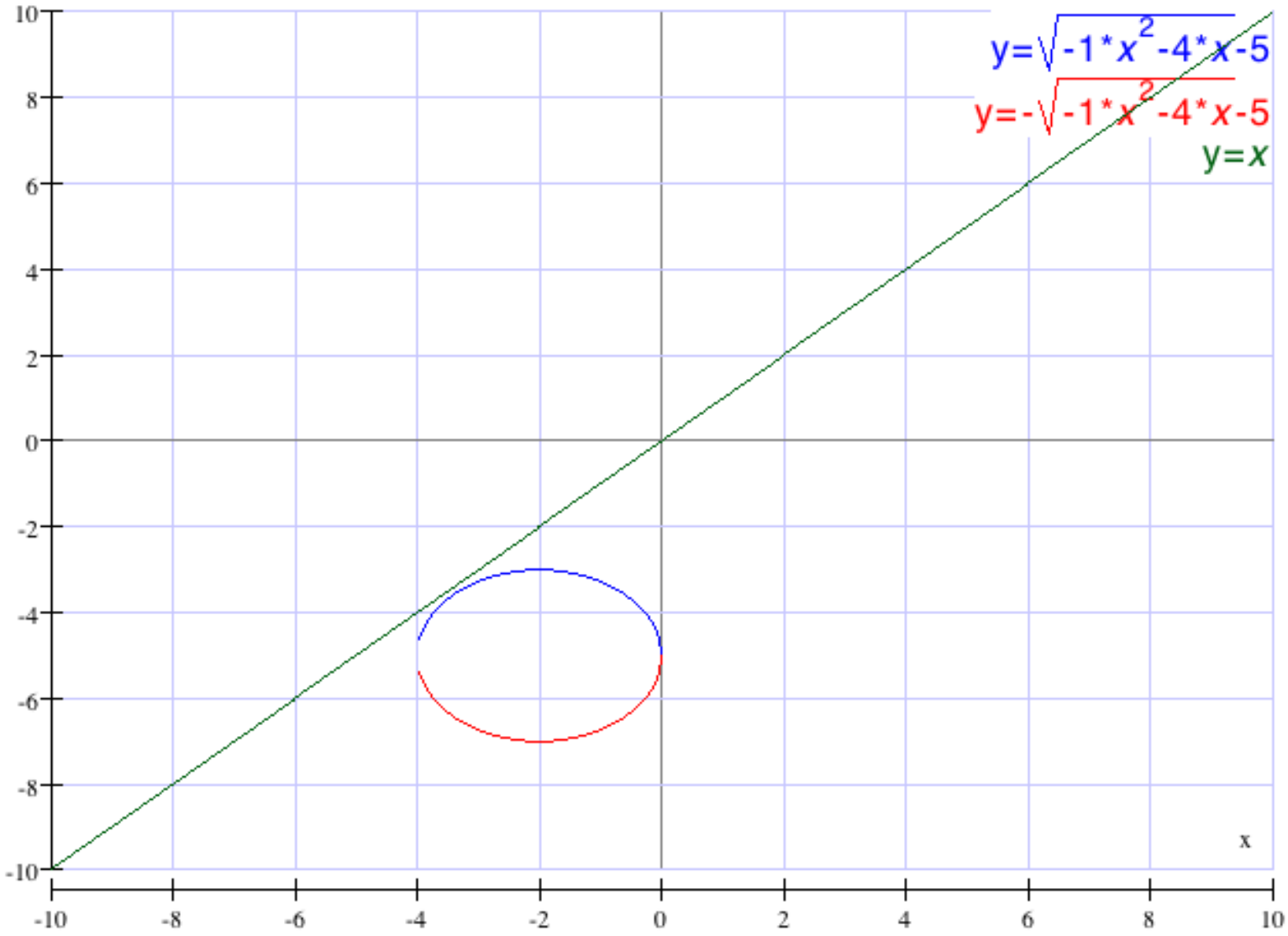
Cosmic Emergency Problem (Source: Froelich, 1996)

You are employed in the Galactic Command Center on the planet Alpha. Alpha's location is at the Galactic coordinates of (-2,-5) and the planet has a radius of 2.

One of Alpha's space stations has just observed a comet at coordinates (50,50). The comet is following a straight -line path and will pass through the Galactic center (0,0) in one week.

Central Command has decided that it is your job to determine whether the comet will collide with Alpha.

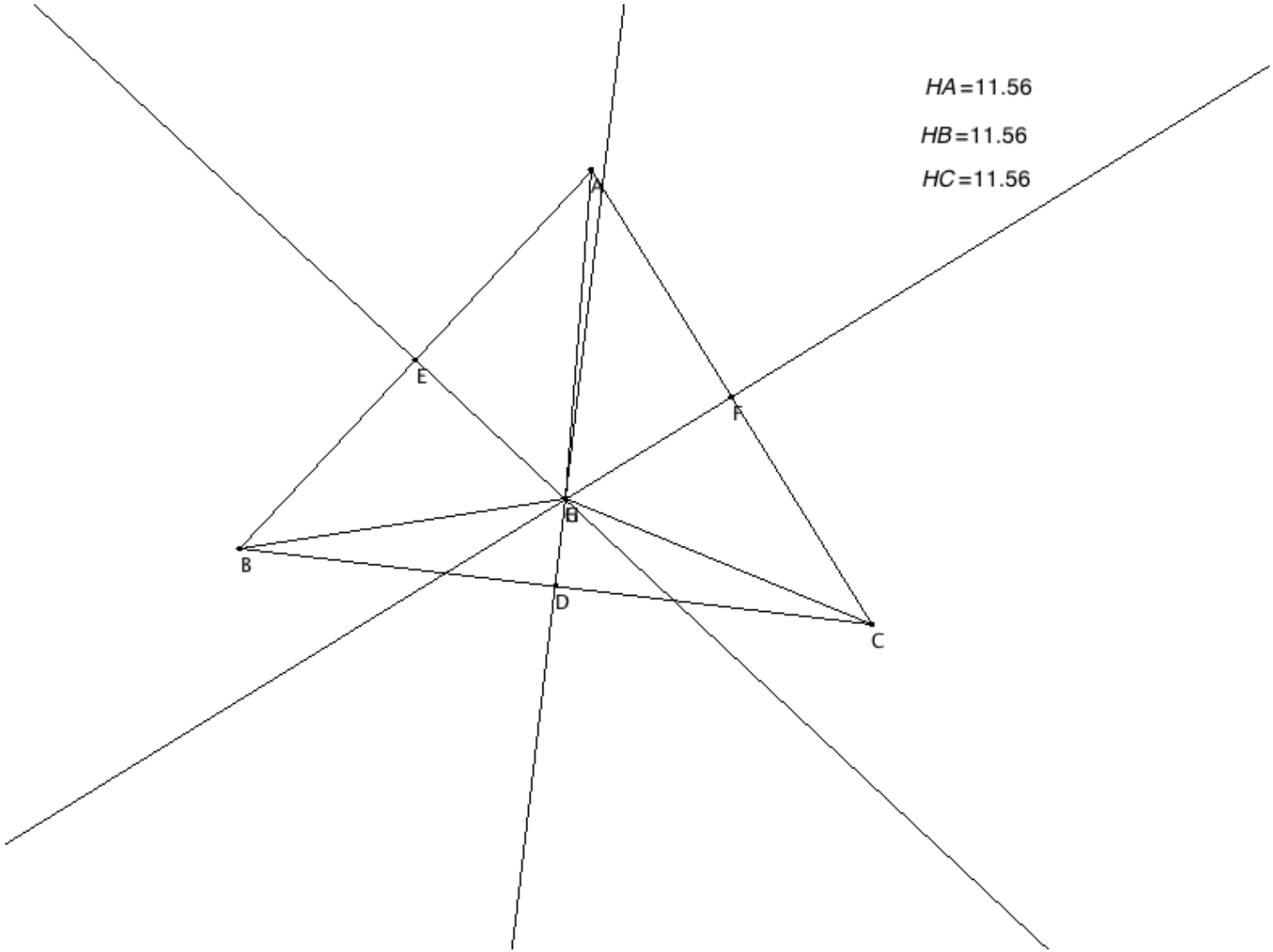
If you decide the comet will not collide with Alpha, you must determine how close it will come. If the comet passes within a tenth of a unit of the planet, there will be some damage to the ecology of the planet.


y=√(-1*x^2-4*x-5)
y=-√(-1*x^2-4*x-5)
y=x
x
10
8
6
4
2
0
-2
-4
-6
-8
-10
-10
-8
-6
-4
-2
0
2
4
6
8
10

Geometry Problem

Given three points, find the point that is equidistant from the three points.

1. How can this problem be reframed in a real-world context?
2. Describe how dynamic geometry software could be used to find the answer.
3. Make and text a conjecture.
4. Prove or disprove your conjecture.


HA=11.56
HB=11.56
HC=11.56
A
E
F
H
B
D
C

Modify the Problem

Given three points, find the point that minimizes the sum of the distances from the point to each of the three given points.

1. How can this problem be reframed in a real-world context?
2. Describe how dynamic geometry software could be used to find the answer.
3. Make and test a conjecture.
4. Prove or disprove your conjecture.

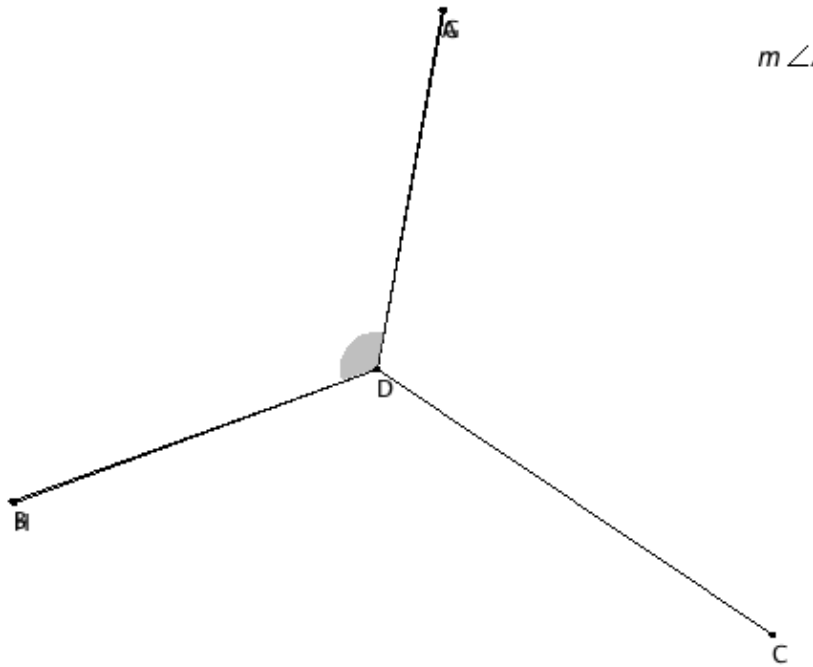
$DA = 10.76$

$DC = 14.09$

$DB = 11.44$

36.3

$m\angle ADB = 120.11$

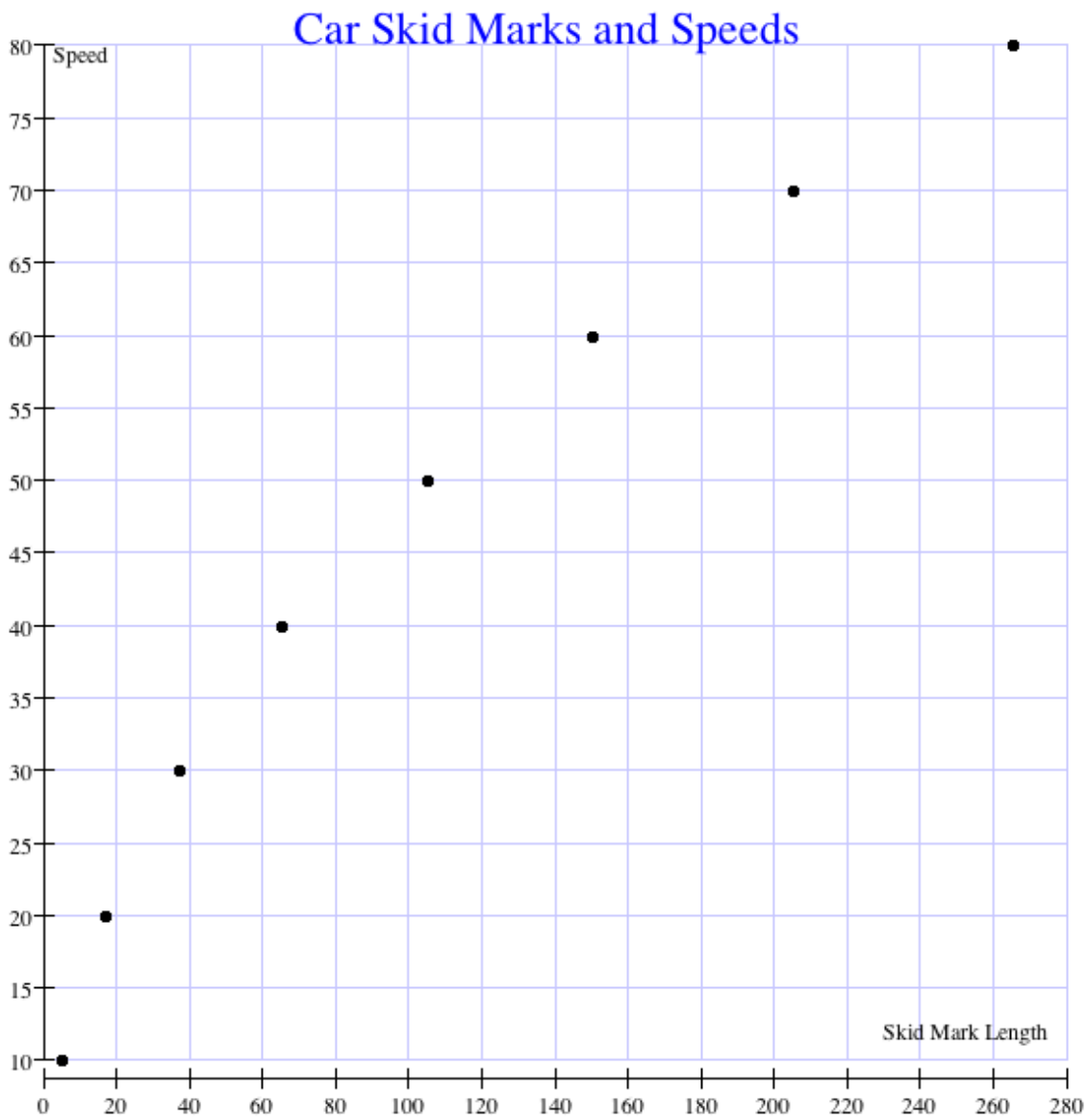


Car Skid Marks and Speeds

When police investigate the scene of an automobile accident, they look for skid marks and use the length of those marks to estimate the speed at which the car was traveling. The results of experiments with a test car, giving skid mark length (in feet) and speed (in miles per hour), are shown in the next slide.

Skid Marks Vs. Speed

	Skid Mark Length	Speed
1	5	10
2	17	20
3	37	30
4	65	40
5	105	50
6	150	60
7	205	70
8	265	80


Car Skid Marks and Speeds
Speed
80
75
70
65
60
55
50
45
40
35
30
25
20
15
10
Skid Mark Length
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280

Simulation Example

During one football game when Aaron was a junior playing for the University of California, he was a perfect 23 for 23, which tied an NCAA record for consecutive pass completions in a game.

With 2 minutes left in the game and California behind USC 23-17, California got the ball, but Aaron threw 3 incomplete passes and was then sacked to end the game.

Aaron entered the game having completed 74% of his passes that season.

Design a simulation to answer the following two questions:

- What is the probability that a passer with a 74% completion rate would have 3 incomplete passes in a row?
- What is the probability that a passer with a 74% completion rate would complete 23 passes in a row?

Technology Assignment

- Part 1: Write a detailed lesson plan that incorporates the use of Core Math Tools with the main objective of the lesson coming from the High School Conceptual Category of Modeling. Since Modeling is found in all of the other Conceptual Categories, you can choose which other category and the **one specific standard** that your lesson will focus on. But it must be one of the modeling standards (remember the *). Your lesson plan should follow the MACSTEP Lesson Plan outline or the MPS lesson plan format.
- Part 2: You will present your lesson to the class. The lesson will be approximately 30 minutes in length.

Student Sample Lessons

Simulation

Donating blood (Source: NCTM Core Math Tools, 2012)

Data Analysis

Drug filtering – exponential decay (Source: NCTM, 2012)

Geometry

Transformations to create motion of a "person"

Development of Pythagorean Theorem

CAS

Investigating the effects of changing parameters a and b in a quadratic function of the form: $y = ax^2 + b$

Summary

Most of my students are at the stage of learning the software and becoming comfortable with the features.

I am hopeful that more of them will use CMT with their students during their student teaching experience this semester.

I am confident that their cooperating teacher will be supportive. The necessary technology is in the schools.

Questions and Comments?

www.nctm.org/coremathtools

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